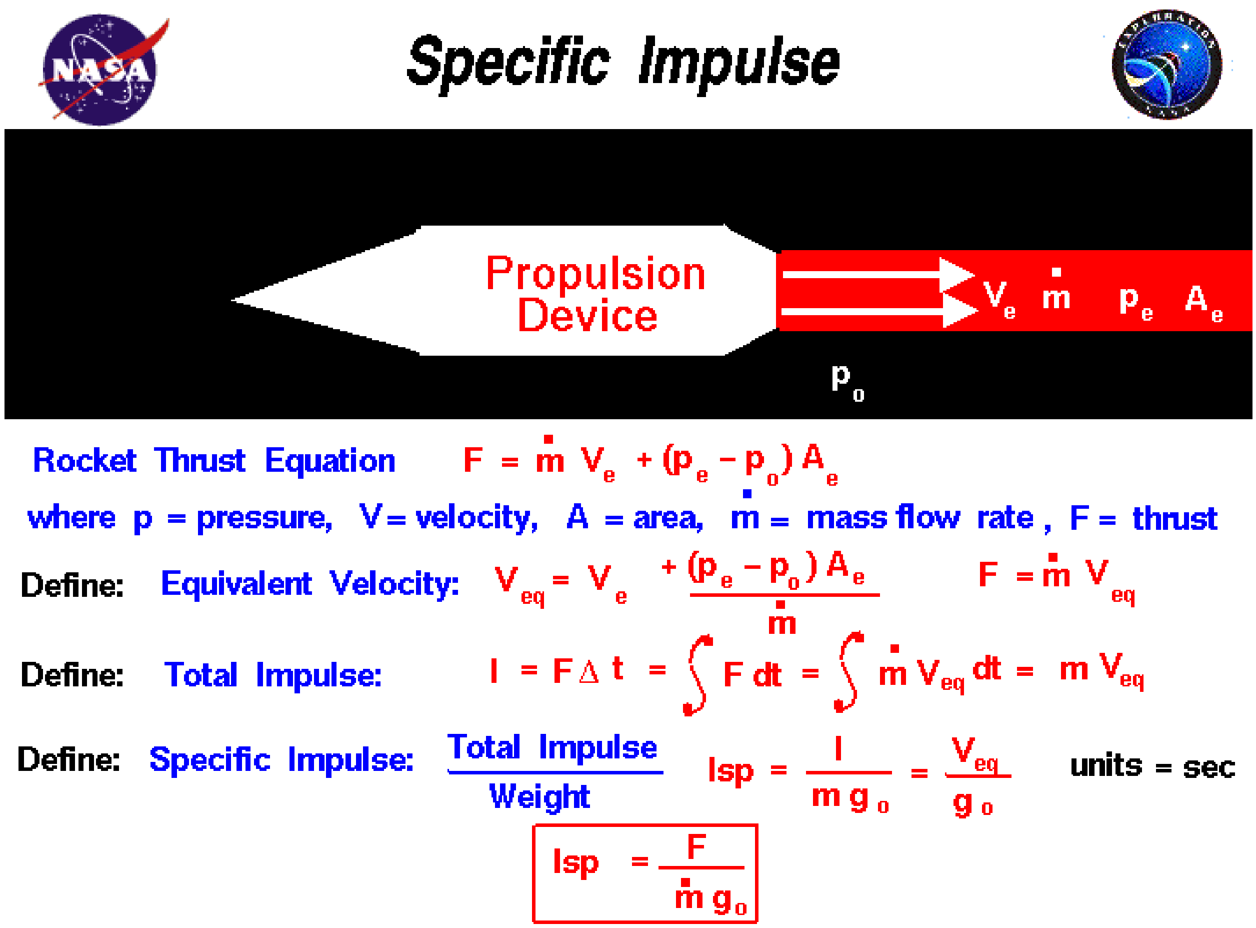
Rocketry – Level 5

For those of who are good at math I have the calculations listed below that are supplied by NASA to calculate the thrust of a model rocket engine. I do understand that there will be few who will be inspired by what is listed here, but I found it interesting and hopefully someone else will also.



Thrust is the force which moves a rocket through the air. Thrust is generated by the rocket engine through the reaction of accelerating a mass of gas. The gas is accelerated to the rear and the rocket is accelerated in the opposite direction. To accelerate the gas, we need some kind of propulsion system. We will discuss the details of the propulsion system on some other pages. For right now, let us just think of the propulsion system as some machine which accelerates a gas.

From Newton's second law of motion, we can define a force to be the change in momentum of an object with a change in time. **Momentum** is the object's mass times the velocity. When dealing with a gas, the basic thrust equation is given as:

F = mdot e * Ve - mdot 0 * V0 + (pe - p0) * Ae

Thrust **F** is equal to the exit mass flow rate **mdot e** times the exit velocity **Ve** minus the free stream mass flow rate **mdot 0** times the free stream velocity **V0** plus the pressure difference across the engine **pe - p0** times the engine area **Ae**.

For liquid or solid rocket engines, the propellants, fuel and oxidizer, are carried on board. There is no free stream air brought into the propulsion system, so the thrust equation simplifies to:

F = mdot * Ve + (pe - p0) * Ae

where we have dropped the exit designation on the mass flow rate.

Using algebra, let us divide by mdot:

F / modt = Ve + (pe - p0) * Ae / mdot

We define a new velocity called the **equivalent velocity Veq** to be the velocity on the right hand side of the above equation:

Veq = Ve + (pe - p0) * Ae / mdot

Then the rocket thrust equation becomes:

F = mdot * Veq

The **total impulse (I)** of a rocket is defined as the average thrust times the total time of firing. On the slide we show the total time as "delta t". (delta is the Greek symbol that looks like a triangle):

I = F * delta t

Since the thrust may change with time, we can also define an integral equation for the total impulse. Using the symbol (Sdt) for the integral, we have:

I = S F dt

Substituting the equation for thrust given above:

I = S (mdot * Veq) dt

Remember that **mdot** is the mass flow rate; it is the amount of exhaust mass per time that comes out of the rocket. Assuming the equivalent velocity remains constant with time, we can integrate the equation to get:

$$I = m * Veq$$

where **m** is the total mass of the propellant. We can divide this equation by the weight of the propellants to define the **specific impulse**. The word "specific" just means "divided by weight". The specific impulse **Isp** is given by:

$$Isp = Veq / g0$$

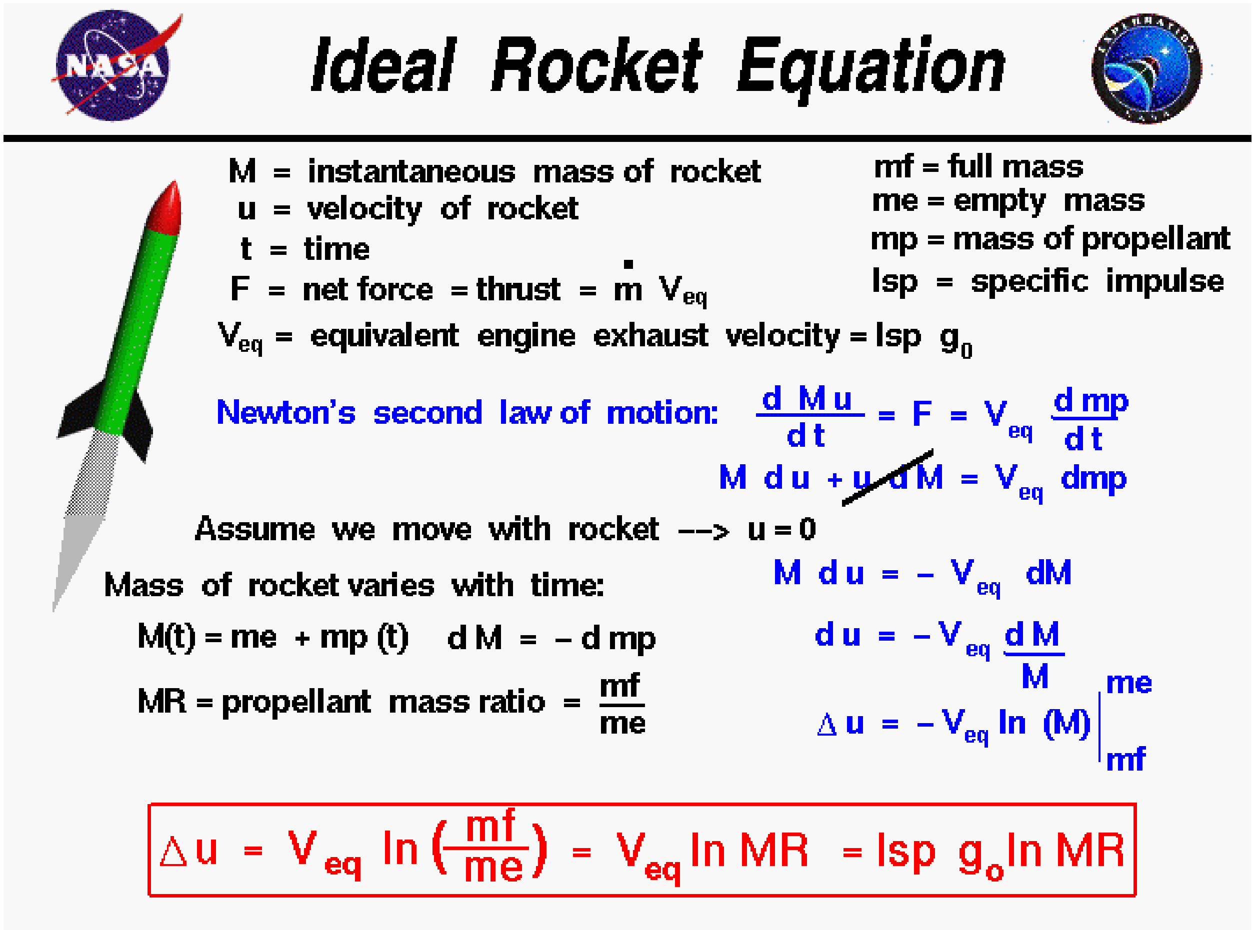
where g0 is the gravitational acceleration constant (32.2 ft/sec^2 in English units, 9.8 m/sec^2 in metric units). Now, if we substitute for the equivalent velocity in terms of the thrust:

$$Isp = F / (mdot * g0)$$

Mathematically, the Isp is a ratio of the thrust produced to the weight flow of the propellants. A quick check of the units for Isp shows that:

$$Isp = m/sec / m/sec^2 = sec$$

Why are we interested in specific impulse? First, it gives us a quick way to determine the thrust of a rocket, if we know the weight flow rate through the nozzle. Second, it is an indication of engine efficiency. Two different rocket engines have different values of specific impulse. The engine with the higher value of specific impulse is more efficient because it produces more thrust for the same amount of propellant. Third, it simplifies our mathematical analysis of rocket thermodynamics. The units of specific impulse are the same whether we use English units or metric units. Fourth, it gives us an easy way to "size" an engine during preliminary analysis. The result of our thermodynamic analysis is a certain value of specific impulse. The rocket weight will define the required value of thrust. Dividing the thrust required by the specific impulse will tell us how much weight flow of propellants our engine must produce. This information determines the physical size of the engine.



The forces on a rocket change dramatically during a typical flight. This figure shows a derivation of the change in velocity during **powered flight** while accounting for the changing mass of the rocket. During powered flight the propellants of the propulsion system are constantly being exhausted from the nozzle. As a result, the weight of the rocket is constantly changing. In this derivation, we are going to neglect the effects of aerodynamic lift and drag. We can add these effects to the final answer.

Let us begin with Newton's second law of motion, shown in blue on the figure:

$$d\,(M\,u)\,/\,dt = F\ net$$

where **M** is the mass of the rocket, **u** is the velocity of the rocket, **F net** is the net external force on the rocket and the symbol **d / dt** denotes that this is a differential equation in time **t**. The only external force which we will consider is the thrust from the propulsion system.

On the web page describing the specific impulse, the thrust equation is given by:

$$F = mdot * Veq$$

where **mdot** is the mass flow rate, and **Veq** is the equivalent exit velocity of the nozzle which is defined to be:

$$Veq = V\ exit + (p\ exit - p0) * Aexit / mdot$$

where **V exit** is the exit velocity, **p exit** is the exit pressure, **p0** is the free stream pressure, and **A exit** is the exit area of the nozzle. **Veq** is also related to the specific impulse **Isp**:

$$Veq = Isp * g0$$

where **g0** is the gravitational constant. **m dot** is mass flow rate and is equal to the change in the mass of the propellants **mp** on board the rocket:

$$mdot = d\ mp / dt$$

Substituting the expression for the thrust into the motion equation gives:

$$d\ (M\ u) / dt = V\ eq * d\ mp / dt$$

$$d\ (M\ u) = Veq\ d\ mp$$

Expanding the left side of the equation:

$$M\ du + u\ dM = Veq\ d\ mp$$

Assume we are moving with the rocket, then the value of **u** is zero:

$$M\ du = Veq\ d\ mp$$

Now, if we consider the instantaneous mass of the rocket **M**, the mass is composed of two main parts, the empty mass **me** and the propellant mass **mp**. The empty mass does not change with time, but the mass of propellants on board the rocket does change with time:

$$M(t) = me + mp\ (t)$$

Initially, the full mass of the rocket **mf** contains the empty mass and all of the propellant at lift off. At the end of the burn, the mass of the rocket contains only the empty mass:

$$M\ initial = mf = me + mp$$

$$M\ final = me$$

The change on the mass of the rocket is equal to the change in mass of the propellant, which is negative, since propellant mass is constantly being ejected out of the nozzle:

$$dM = - d\ mp$$

If we substitute this relation into the motion equation:

$$M\ du = - Veq\ dM$$

$$du = - Veq\ dM / M$$

We can now integrate this equation:

$$delta\ u = - Veq\ \ln (M)$$

where **delta** represents the change in velocity, and **ln** is the symbol for the natural logarithmic function. The limits of integration are from the initial mass of the rocket to the final mass of the rocket. Substituting for these values we obtain:

$$delta\ u = Veq\ \ln (mf / me)$$

This equation is called the **ideal rocket equation**. There are several additional forms of this equation which we list here: Using the definition of the propellant mass ratio **MR**

$$MR = mf / me$$

$$delta\ u = Veq * \ln (MR)$$

or in terms of the specific impulse of the engine:

$$delta\ u = Isp * g0 * \ln (MR)$$

If we have a desired delta u for a maneuver, we can invert this equation to determine the amount of propellant required:

$$MR = \exp (delta\ u / (Isp * g0))$$

where **exp** is the exponential function.

If you include the effects of gravity, the rocket equation becomes:

$$delta\ u = Veq\ \ln (MR) - g0 * tb$$

where **tb** is the time for the burn.